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# High Frequency Electromagnetic Propagation/Scattering

## STTR Contract F49620-98-C-0042

## Final Technical Report


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|---|---|
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# Preface

The work presented in this report was performed at Matis, Inc., 1255 Biltmore Drive, Atlanta, Georgia 30329 and at the Department of Mathematics, University of California, Los Angeles, CA 90095-1555. This work was sponsored by the Air Force Office of Scientific Research during the period September 1, 1998 - August 31, 2000. The project Technical Monitor was Dr. Arje Nachman from the AFOSR. We are indebted to Dr. Nachman for all the encouragement and support he gave us during the work on this contract.

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# 1 Executive Summary

Two directions of work have been pursued under this effort. The first direction is concerned with further extensions of our geometric techniques for spatial and surface ray tracing and applications of these techniques to problems in high-frequency electromagnetics. In addition, high-frequency physical optics methods were developed for accurate and efficient calculation of scattered fields due to currents in the penumbra region. The second direction is concerned with development of grid methods for capturing the motion of self-intersecting interfaces and related PDEs connected with propagation of high-frequency waves. In both directions significant progress has been achieved. For convenience of reference these two directions are designated below as **Efforts A and B**.

**Under effort A,** the theoretical work carried out by the Matis, Inc. team resulted in new methods for solving global problems of determining geodesics in presence of obstacles with highly complex geometries. This is a critical capability necessary for finding accurate and reliable high-frequency solutions to numerous scattering problems. The theoretical work was accompanied with a very extensive developments of computational methods.

In particular, the theoretical results obtained under effort A were implemented into a system of computational codes integrated into two fully operational computer software systems for interactive analysis of airborne antenna-to-antenna electromagnetic interference/compatibility and interactive analysis of airborne antenna radiation patterns. Both computer systems have truly revolutionary capabilities to perform antenna analysis on realistic digital aircraft models represented by models produced with commonly used computer aided design (CAD) methods. Even though the development of these systems has only recently been completed, a number of inquiries have been received from the Air Force and commercial companies working on defense contracts with the US Government. Such interest in the developed systems is strong evidence that the results of this work are applicable to solution of real-life problems important for the US defense.

In addition, under the effort A the Matis team developed a very efficient and stable method for computing the correction terms to the physical optics (PO) fields due to currents at the shadow boundary of a scatterer. This method will be applied to enhance the accuracy of radar cross sections (RCS) predictions by PO methods.

**Under effort B,** the team at UCLA developed a revolutionary fixed grid method for capturing the motion of self-intersecting interfaces and related PDEs. Moving interfaces that self-intersect arise naturally in the geometric optics model of wavefront motion. Ray tracing techniques can be used to compute these motions, but they lose resolution as rays diverge. This difficulty was overcome by developing a new numerical method that maintains uniform spatial resolution of the front at all times. The approach is a fixed grid, interface capturing formulation based on the Dynamic Surface Extension method of Steinhoff and Fan. The new methods can treat arbitrarily complicated self-intersecting fronts, as well as refraction, reflection and focusing. We also further extended this approach to curvature dependent front motions, and the motion of filaments. The new methods have been validated with numerical experiments.

In summary, both directions of work were complementary to each other and produced significant theoretical results and numerical codes with capabilities for solving important

and difficult high-frequency wave propagation problems.

We describe now the results in more detail.

# 2 Effort A

## 2.1 Geometric Techniques in High-Frequency Computational Electromagnetics

An important practical problem in communications is to numerically predict radiation directivity properties of antennas mounted on geometrically complex platforms acting as scatterers. High-frequency methods of electromagnetics have been used extensively for dealing with this problem in cases when the platform can be represented as a collection of a small number of simple shapes such as cylinders, ellipsoids, pieces of planes, cones, etc. When such a representation of the platform is sufficiently accurate the high-frequency methods provide an accurate estimate of the electric field produced by an antenna in a given direction. However, comparisons of computed results with experimental data showed that large errors may result in from inaccurate representation of the platform geometry. Consequently, there is a need for algorithms capable of performing high-frequency analyses in geometrically complex environments that can not be reduced to small collections of simple shapes.

High-frequency ray methods are based on geometrical optics and its extensions and, in order to apply these methods, one needs first to construct the propagation paths from the antenna to a given observation point at infinity specified as a point on a unit far-sphere*. Such observation point at infinity is usually referred to as the "far-field" direction. When the scatterer does not obstruct the wave propagation the path is a linear ray. Otherwise, the scatterer acts as an obstacle and the propagation path may have a nonempty contact set with the surface of the scatterer. In the present setting the scatterer is assumed to be perfectly conducting, and no penetration of the paths through the scatterer is allowed.

One of the main principles of the classical geometrical optics is that fields propagate along paths that are critical points of the Fermat functional. The Euler equations for these critical points lead to the eikonal equation. In classical setting the scatterer is smooth and usually only the paths that propagate in free space and reflect or refract are considered.

In contrast, in the geometric theory of diffraction and its extensions (especially, the uniform theory of diffraction) the notion of a propagation path is generalized to include paths not only with spatial segments and reflections, but also those that diffract at geometric singularities such as corners and edges, may attach to the surface of the scatterer, "creep" along it, then detach, diffract, and so on.

If a propagation path from the source to an observation point is known, the field at the observation point due to that path can be obtained by "gluing" together the local scattering information along the path. In principle, this scattering information can be computed by high-frequency techniques, though application of these techniques is by no

*Typically, an engineer may specify a set of sample directions ("pattern cut") in which the electric field produced by an antenna is required.

means straight-forward. Finally, the total field at the observation point is obtained as a vector sum of contributions by the fields propagating along individual paths from the source to that observation point.

The geometrical optics theory initiated by J. Keller and developed continuously since 1950 focused primarily on local asymptotics of diffracted fields, without providing tools for identifying and computing(!) propagation paths in global, that is, on entire platforms. In present work, a theory of globally defined critical paths of the Fermat functional was developed in the case of polyhedral surfaces. This case is important because in practice scatterers are often represented in this form and, furthermore, for such scatterers we succeeded in constructing efficient computational methods for determining these paths.

The corresponding variational theory associated with the Fermat functional had to be re-examined and extended to include non-classical cases required by diffraction theory. In certain cases, in order to capture paths with specific properties the variations may have to be restricted. For example, in order to obtain paths that contain a reflection point on the scatterer one should require that admissible paths have a non-empty contact set on the scatterer. Similarly, in order to obtains paths diffracting at corners or, in some special cases at edges, one needs to further restrict classes of admissible variations. The definitions of stationary paths in such cases must be modified accordingly.

In applications related to directivity properties of antennas the diffracted field along creeping segments of a propagation path within the shadow region decays exponentially. Therefore, it is important to construct only the "dominant" paths and avoid searching for paths with contributions to the electric field below some user-specified level.

Under these effort, algorithms for building the dominant paths in the discrete radiation problem have been developed and implemented. The algorithm starts with building a possibly large set of initial paths that are quasi-geodesics in the metric of the scatterer. The initial paths are terminated at the shadow boundary (relative to the current far-field direction). For a given far-field direction the algorithm uses the local optimality condition at the shadow boundary to filter out some of the initial paths. In "generic" cases this procedure filters out most of the initial paths. The initial paths that remain are optimized. The optimization of each of the paths uses a a variant of unfolding strategy combined with lifting of paths off the surface of the scatterer whenever possible.

Extensive numerical validations of the codes were performed and the results were compared with available measured data. Very good agreement between the numerical results and the measured data has been established.

## 2.2 Methods for Calculating Scattering Due to Currents at Shadow Boundaries

In typical physical optics (PO) approximations used for computing the scattered far field the surface of the scatterer is divided into the lit and shadow regions. The PO current induced on the scatterer is calculated as

$$J^{PO} = \begin{cases} 2n \times H^i & \text{in the lit region} \\ 0 & \text{in the shadow region,} \end{cases}$$

where $n$ is the normal field on the scatterer and $H^i$ is the incident magnetic field. The fact that the $J^{PO} = 0$ in the shadow region impacts negatively the accuracy of calculated radiated fields, since the contribution from the current in the shadow region is not taking into account the creeping waves propagating into that region. Neglecting this contribution may lead to serious errors especially in cases where bistatic radar cross-sections (RCS) are required.

The ILDC (incremental length diffraction coefficients) method has emerged during the last two decades as a technique that in certain cases permits calculation of of a correction to PO field accounting for edge diffraction for arbitrary incidence and scattering angles. However, the edge diffraction problem has a very specific structure that makes it particularly suitable for treating by the ILDC method. Nevertheless, it seems natural to examine the applicability of the ILDC method to the case when the shadow boundary on a surface is treated as a diffraction line. This approach has been analyzed by A. Yaghjian, R. Shore, and M. Woodworth for the case of the sphere and, more recently, by A. Yaghjian and R. Shore for the case of an ellipsoid of revolution.

The objective of our work was to extend the ILDC method to the case of arbitrary convex surfaces represented by CAD (computer aided design) - generated models. Several serious difficulties have to be overcome when dealing with general surfaces. First of all, in the case of an edge (or an ellipsoid), the edge (shadow boundary in case of the ellipsoid) is well defined, geometrically simple, and can be described analytically. On general surfaces there is no simple description of the shadow line and such a description has to be built by numerical means. Furthermore, the geometric information regarding the surface in the shadow zone also has to be obtained numerically. Secondly, in contrast with the edge diffraction problem, in the case of the diffraction by shadow boundary the starting points of the diffracted rays occupy the whole area of the shadow region reached by the creeping waves. Thirdly, in the case of edge diffraction there are procedures that allow to reduce the computation of the radiation integral to a line integral. Such procedure, in general, is not available for the shadow boundary problem and needs to be developed.

In our approach, developed jointly with Dr. Arie Michaeli from Israel .ve have been successful in recasting the ILDC method in a form independent of a specific surface (such as sphere or ellipsoid) which makes the method applicable to arbitrary convex surfaces. Further, suitable procedures for determining the ILDC's from far fields scattered by canonical problems have been developed. We have now developed an algorithm that can run on fairly general convex surfaces and we are testing it in various special cases. The first results look very good and are encouraging. Once this stage is complete and stable performance of the algorithms is achieved, we can start working on building codes which can work on models represented by faceted and/or NURBS (Non-Uniform Rational B-Splines) surfaces. The machinery that we built for doing geometric computations during Phase I and our other work on scattering problems is a key to this development. An important application for this work is in the area of RCS predictions. In particular, the current version of the widely used RCS code X-patch does not have the capability to account for the fields due to creeping waves in the shadow region. We plan to explore possible options for adding on such a capability to the X-patch code with the current developers of X-patch.

# 3 Effort B

## 3.1 A Fixed Grid Method for Capturing The Motion of Self-Intersecting Interfaces and Related PDEs

We have developed a revolutionary fixed grid method for capturing the motion of self-intersecting interfaces and related PDEs. We outline the method and results here.

Moving interfaces that self-intersect arise naturally in the geometric optics model of wavefront motion. Ray tracing techniques can be used to compute these motions, but they lose resolution as rays diverge. We have developed a new numerical method that maintains uniform spatial resolution of the front at all times. Our approach is a fixed grid, interface capturing formulation based on the Dynamic Surface Extension method of Steinhoff and Fan, "Eulerian computation of evolving surface curves and discontinuous fields", Technical Report UTSI, Tullahoma, TN. The new methods can treat arbitrarily complicated self intersecting fronts, as well as refraction, reflection and focusing. We also further extended this approach to curvature dependent front motions, and the motion of filaments. We validated the new methods with numerical experiments.

In the limit of short wavelengths, it is well known that a wavefront moving through a medium can be described as a moving surface with a normal velocity that depends on position,

$$\vec{v} = c(x)\hat{n}$$

where $\hat{n}$ is the local normal to the front and $c(x)$ is the local wave speed. Notable examples include the short wavelength approximation of seismic and electromagnetic pulses, as well as the familiar example of ripples moving on the surface of a pond. An important feature of this idealized wavefront motion is that intersecting wavefronts pass through each other, and also that they reflect and refract off boundaries.

Many interesting numerical methods have been developed to compute these complex motions. The most detailed approach is to discretize the governing wave equations directly. Unfortunately, this approach is often impractical because it requires that the discretization resolve the short wavelengths, which may be thousands of times smaller than the length scale of interest.

At the other extreme, ray tracing can be used to evolve waterfronts according to geometrical optics. here, the front is represented using a number of markers which are moved independently. This approach has the advantage of simplicity, but the markers tend to diverge which leads to loss of resolution and aliasing of the front.

To maintain a uniform resolution of the interface, it is natural to consider a fixed grid, interface capturing formulation such as the Level Set method. Here, the wavefront is represented as the zero contour of a smooth function $\phi$, which in turn evolves according to the level set equation

$$\phi_t + c(x)|\nabla\phi| = 0.$$

This can be solved accurately and efficiently using numerical Partial Differential Equation (PDE) techniques. Unfortunately, the basic level set method is inappropriate for treating evolving wavefronts because the solutions to this PDE will have fronts merge upon colliding, rather than pass through one another.

To obtain a fixed grid method appropriate for capturing wavefront self-intersection Steinhoff and Fan proposed Dynamic Surface Extension (DSE) methods. These schemes start from some spatially distributed representation of the interface (similar to, but more general than, the level set $\phi$ representation), and the motion is achieved by alternating between two steps: a simple short time evolution comparable to ray tracing, and an extension step that updates the distributed representation to reflect the new front location. DSE methods automatically give a uniform resolution of expanding fronts by using a fixed spatial grid, and the fronts automatically pass through one another rather than merging.

The original DSE methods are not well suited to certain fundamental self-intersection problems such as the formation of swallowtails. We have generalized a basic DSE scheme (the Closest Point Method) to handle this fundamental problem, as well as all other complex intersections. We further generalized our approach to reflecting and refracting wavefronts. We also discussed new extensions for propagating intensity values, for treating curvature-dependent flows, and for treating the motion of filaments (or more generally, objects of co-dimension $> 1$).

# The Closest Point Method

*Initialize.* For each point $x \in R^n$: Set the initial tracked point $TP(x)$ equal to the closest point (to $x$) on the initial surface $\Lambda_0$. set $\hat{n}$ equal to the surface normal at the tracked point $TP(x)$, and let $c$ denote the wavefront speed at the tracked point.

Repeat for all steps:

1. *Evolve* the tracked point $TP(x)$ according to the local dynamics for a time $\Delta t$ : $TP(x)_t = c\hat{n}$.

2. *Extend* the surface representation by setting each tracked point $TP(x)$ equal to the true closest point on the updated surface *Gamma*, where $\Gamma$ is defined to be the locus all tracked points, i.e., $\Gamma = \{TP(x) | x \in R^n\}$. Replace each $\hat{n}(x)$ by the normal at the updated $TP(x)$.

End.

Intuitively, the manner in which this method treats self-intersection is most easily understood by considering how it treats two colliding, planar waves. Initially, each nodal tracked point value is set equal to the closest point on the nearest wavefront. These tracked points are updated during the Evolution Step according to $TP(x)^{new} = TP(x)^{original} + c\hat{n}\Delta t$. Notice that the updated tracked points are no longer the true closest points. Finally, the Extension Step resets each nodal value to be a true closest point.

# Implementation

In practice, the Initialization Step of the Closest Point Method can often be handled analytically in simple problems. More complicated wavefronts can be treated using fast tree-based algorithms. Implementation of the evolution Step is also straightforward because

each tracked point is just updated according to $TP(x)^{new} = TP(x)^{original} + c\hat{n}\Delta t$. The final Extension Step is more complicated and is typically divided into two parts, a search step and an interpolation step.

In the search step, the updated value fro a node is taken to be the closest of all tracked points (localization of this step is possible). This gives an improved approximation of the closest point representation. Unfortunately, this process cannot create any new tracked points so diverging wavefronts will lose resolution. Thus, a second interpolation step is needed in order to maintain a uniform representation.

Steinhoff, Fan and Wang carry out this interpolation by averaging over nearby nodes. This very simple approach is effective for a variety of interesting problems, but it can produce spurious wavefronts in certain cases and is low order accurate. For these reasons, we consider a higher order interpolation based on nearby neighbors. These neighbors (call them $y$ and $z$) are chosen so that $x, y$ and $z$ are collinear and roughly parallel to the interface. If the tracked points for $x$ and $y$ are distinct and lie on the same smooth curve, then an improved estimate for the closest point to $x$ can be generated using the nodal values at $x$ and $y$. Similarly, an improved closest point estimate can be attempted using the nodal values at $x$ and $z$. The closest of these two results to $x$ is taken to be the updated nodal value.

Notice that this Extension Step does not yield a true closest point representation. However, closest point values are expected *near* the interface. Furthermore, this extension has the useful property that every nodal value represents some tracked point on the interface.

# First Arrival Times

Unfortunately, the Closest Point Method can produce gaps in the interface. We now discuss a method which gives a much more uniform representation of the interface.

The formation of gaps for the Closest Point Method is most easily understood by considering how swallowtails are represented. When the swallowtail is small, nodal values over-represent corners. Since large regions are used to represent corner points, few grid points are available to represent the end of the swallowtail. This uneven treatment leads to gaps in the interface which propagate and grow.

To obtain an improved result, a more uniform representation is needed. For example, we can set each nodal value to be the point on the interface with the *minimal arrival time* rather than the minimal distance. In a homogeneous Medium, without reflection, this just means that each nodal value is set equal to the closest point on the interface that propagates directly to or from the node. Using this representation, we find that redundancy is largely eliminated, and a greatly improved approximation of the swallowtail is obtained. Unfortunately, arrival times are often difficult and expensive to evaluate in the variable index of refraction case or when reflections occur. Furthermore, even in a homogeneous medium, this approach requires a more intricate search step nodal values can only be updated when a nearby tracked point travels directly towards the node (which rarely occurs).

Of course, we are not limited to representations that minimize distance or arrival times – a minimization based on some combination of distance and direction of motion can also

be carried out. A particularly interesting choice arises when nodal values are set equal to the "Minimizing Point"

$$MP(x) = \min_{y \in \text{Interface}} \gamma |(x - y) \cdot \hat{n}^{\perp}(y)| + \|x - y\|^2 \tag{1}$$

for $\gamma > 0$ since then a good agreement with the minimal arrival time representation is found near the interface. Using this idea leads to the following modification of the Closest Point Method:

## The Arrival Time Method

*Initialize.* For each point $x$: Set the tracked point $TP(x)$ equal to the minimizing point $MP(x)$ on the initial surface $\Gamma_0$ for the minimization in Eq. (1). Set $\hat{n}$ equal to the normal at $MP(x)$.

Repeat for all steps:

1. *Evolve* the tracked point $TP(x)$ according to the local dynamics for a time $\Delta t$ : $TP(x)_t = c\hat{n}$.

2. *Extend* the surface representation by replacing each tracked point $TP(x)$ by the point $MP(x)$ on the updated surface $\Gamma = \{TP(x) | x \in R^n\}$ that minimizes Eq. (1). Replaces each $\hat{n}$ by the normal at the updated $MP(x)$.

End.

This simple approach gives a much more uniform representation and naturally treats the prototype swallowtail problem.

We have localized the method and applied it to problems involving refraction and reflection. We have also calculated the intensity by using wave front curvature. The results were very good.

## The Segment Projection Approach

Another approach to this problem was taken by A.-K. Tornberg, O. Runborg,and B. Engquist. This is based on the "segment projection method" where (a) each interface $\gamma$ is given as a union of curve segments $\gamma_j$. (b) The segments are chosen so they can be represented by a function of one coordinate variable. (c) The domain of the independent variables of these functions are projections of the segments onto the coordinate axes.

In addition to discretization of these functions one needs information about the connectivity of the segments. For each part of an $x$-segment, one needs to define which $y$-segment it corresponds to, and vice versa.

Each of these functions is updated according to a simple grid based advection equation.

This technique has been very successfully applied to a host of variable index of refraction, two dimensional problems. Segments cross each other without merging. Swallowtails and other singularities can be handled by the use of phase plane variables.

# 4 Publications

**Effort A.** Some of the computational aspects of this work related to electromagnetic compatibility were presented in the paper by P. Hussar, V. Oliker, H. Riggins, E. Smith-Rowland, W. Klocko, and L. Prussner, "An implementation of the UTD on facetized CAD platform models", IEEE Antennas and Propagation Magazine, vol. 42, no. 2, April 2000, pp.100-106.

Some of the theoretical results obtained under this effort were presented in the paper by V. Oliker, "Electromagnetic scattering, 1D/2D obstacle problem, and discrete geodesics", Proceedings of the 16-th European Workshop on Computational Geometry, EURO-CG-2000, March 2000, pp. 66-69.

An exposition of some of the computational methods and validation results were also included in the report "DOVA - A computational system for analysis and prediction of Antenna-to-Aircraft and Antenna-to-Antenna Interactions", April 2000, pp. 1 - 219, submitted by Matis, Inc. to the AFRL, Rome, NY.

Computational software systems implementing developed methods are described in (and can be downloaded from):

- "DOVA - A computational system for real time analysis and prediction of radiation patterns of antennas mounted on platforms represented by CAD models", http://www.matis.net, and
- "C-DOVA - A computational system for real time analysis and prediction of EM interactions between antennas mounted on platforms represented by CAD models", http://www.matis.net

In addition, detailed expositions of theoretical and computational results obtained under this effort are still being prepared for publication.

**Effort B.** The results obtained under this effort are reported in a UCLA CAM report #99-22, and is joint work by Steven J. Ruuth, Barry Merriman and Stanley Osher. This report may be downloaded from http://www.math.ucla.edu/applied/cam/index.html. The paper has also appeared in the Journal of Computational Physics, v.163, (2000), pp. 1-21.